Contact layer as a propelling advantage in throwing

Franck Celestini,1 Joachim Mathiesen,2 Médric Argentina,1 and Christophe Raufaste1

1 Université Côte d’Azur, CNRS, Institut de Physique de Nice, 06100 Nice, France
2 Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

The ability to throw objects is important to hominid evolution and has been central to many technological innovations in human history. The conditions for optimal throwing performance, however, are in general unknown. Here we present ejection experiments of projectiles composed of a rigid and a soft layer. The optimal performance is achieved when the soft layer of the projectiles matches the acceleration dynamics of the throwing engine. Compared with a fully rigid projectile, a bi-layered projectile with a small soft elastic layer can achieve a more than 300% increase in translational kinetic energy. Our study emphasizes the importance of not only designing projectiles to optimize the stability in flight, the aerodynamic profile and the spin, but also to carefully match the mechanical impedance of the projectiles and their thrower.

I. INTRODUCTION

In humans and animals, the release of energy stored inside elastic fibrous structures, like tendons, can produce power outputs far beyond the capability of mere muscles [1]. In fact, elastic tissue structures recoil faster than muscles can shorten, which is used, for example, when jumping [2–5], running [6, 7] or throwing [8, 9]. The ability to throw objects has offered an evolutionary advantage to primates [10] and in particular humans [11–13], and is tightly coupled to the elasticity of the shoulder [8]. The mechanical work produced in a throw starts with a motion of the hips, is followed by a loading of the elastic elements in the shoulder, and ends with a rapid kinetic energy transfer to the arm and projectile [9]. Underlying these steps, different muscles (in the legs, hips, torso, shoulder, elbow and wrist) are activated sequentially and the optimal throw happens when the different joints are put into motion with carefully timed delays [14–17]. The timing and activation of successive segments is an active subject of research in the field of robotics, biomechanics, sports and human evolution [8, 9, 18–24]. The idea of precise synchronization and timing in throwing is also important in plants and fungi, which use throwing-like mechanisms to release and disperse spores and pollen [25, 26]. For example, the sporangia - the spore ejection units in ferns - take advantage of two different time scales to maximize the ejection [27]. Finally, throwing experiments with soft elastic objects have shown that the transfer of energy can be optimized, up to 250%, when the ratio of the time scale of acceleration to the projectile eigenperiod takes a particular value [28].

Inspired by these mechanisms, we design a projectile which delays the release of elastic energy and optimizes the energy transfer within a chain of elastic elements. Specifically, we consider bi-layered projectiles thrown by an ejection engine. We show that the throw efficiency of rigid objects, which is generally low, can be drastically improved by more than 300% by adding an extra soft layer between the engine and the rigid object.

FIG. 1. (A) Basic description of the ejection experiment: initial configuration and takeoff at velocity $V_e$. (B) Maximal height reached by three projectiles, E1, E2 and E3, with different rigid/soft ratios: fully rigid, a small soft layer and a large soft layer, respectively. The engine frequency is the same in all cases, $f = 76$ Hz. (C) Image sequences (0.7 ms between images) of these three experiments. For each sequence, an arrow denotes the takeoff time. By using two polarizers, we observed coloring inside the soft material: the photoelastic properties of the hydrogel provided information about the local deformation. The length scale is given by the diameter of the projectiles, 12 mm in all cases. Movies available in the Supplemental Material [29].

II. MATERIALS AND METHODS

A. Ejection setup

The experiments were performed with the ejection engine illustrated in Fig. 1A. From $t = 0$, the engine plate performs a uni-axial harmonic motion described to a high precision by the function $z_p(t) = A[1 − \cos(2\pi ft)]$. The frequency $f$ of the engine can be varied in the range 20 - 80 Hz and the amplitude $A$ is approximately 1 mm. Hence, the maximum velocity of the engine, $V_p^* = 2\pi fA$, is around 0.5 m.s$^{-1}$ and the maximum acceleration is around 25 g, where $g$ is the acceleration of gravity. More-
over, the weights \( m \) of the projectiles are negligible relative to the weight of the engine and therefore do not influence the engine motion. We use cylindrical projectiles with a diameter of 12 mm and a length \( L \) of 15 mm. The projectiles consist of a soft and a rigid material superposed. The rigid elastic material is made of a hard plastic (polyether ether ketone, Young modulus \( E_r = 3.6 \) GPa and density \( \rho_r = 1300 \text{ kg.m}^{-3} \)) and the soft elastic material is made of a gelatin hydrogel (\( E_s = 12 \) kPa and \( \rho_s = 1000 \text{ kg.m}^{-3} \)). Hydrogels were prepared by dissolving gelatin powder (Sigma-Aldrich, gelatin from porcine skin, gel strength 300, type A) in water for 30 min at 80°C under gentle agitation. The ratio of powder/water is around 15% in mass. In our range of parameters, the hydrogels behave mostly elastically and viscoelastic effect can be neglected. To prepare each projectile, the rigid part was placed at the bottom of a cylindrical mold where the dissolved solution was poured and kept at rest for two hours. Once unmolded the hydrogel has reticulated and is both elastic and transparent. The contact with the rigid layer is cohesive and prevents the two parts from separating. Gelatin hydrogels are photoelastic, a property exploited to track the local deformation inside the projectiles. The ejection efficiency is quantified after takeoff by the energy transfer factor \( \alpha = (V_e/V_p)^2 \), where \( V_e \) is the ejection velocity of the center of mass. This quantity is calculated from the maximal height \( H \) of the free flight following complete ejection (\( V_e = \sqrt{2gH} \)).

We present in Fig.1B,C the ejection dynamics for three typical projectiles: (E1) a 100% rigid object, (E2) a bi-layered object with a small soft layer at the bottom, and (E3) one with a large soft layer at the bottom. For this engine frequency, \( f = 76 \) Hz, the projectile with the small soft layer is thrown significantly faster—and is ejected higher in consequence—than the purely rigid projectile and the projectile with the large soft layer. An optimum in the soft/rigid ratio is therefore expected. Observing the photoelastic coloring inside the soft layer (Fig.1), we note that a deformation wave propagates inside this layer, and that elasticity needs to be accounted for to understand the dynamics.

**B. Simulations**

The experiments are supported by numerical simulations of the deformation dynamics. For the sake of generality, we consider the wave propagation in both the soft and rigid layers. In cylindrical coordinates and assuming axial symmetry of the projectiles, the displacement \( \mathbf{u} = (u_r, u_\theta, u_z) \) has the components \( u_\theta = 0, u_r = u_r(r, z, t) \) and \( u_z = u_z(r, z, t) \). We do not expect the longitudinal displacement \( u_z \) to depend significantly on the radial coordinate \( r \), so we approximate \( u_z(r, z, t) \approx u(z, t) \). The dynamics are therefore driven by the propagation of longitudinal elastic waves inside the projectile. The waves are described by the one-dimensional momentum balance equation,

\[
\rho(z)\partial_z^2 u(z, t) - \partial_z [E(z)\partial_z u(z, t)] = 0, \tag{1}
\]

where \( z \) measures the distance from the bottom part of the cylinder along the symmetry axis, \( E(z) \) is the local Young modulus and \( \rho(z) \) the mass density. For our bi-layered projectiles, the material parameters are constant in each layer and we recover the classical wave equation with two longitudinal wave velocities, \( c_s = \sqrt{E_s/\rho_s} \) and \( c_r = \sqrt{E_r/\rho_r} \), inside the soft and rigid layers, respectively. We use the sub-indices \( s \) and \( r \) to denote physical quantities of the soft and rigid materials, respectively. For the numerical simulations we used a basic semi-implicit (symplectic) Euler finite difference scheme. The point of contact with the plate is assumed to be fully rigid such that the end of the rod upon starting immediately assumes the speed of the plate. The dynamics are subject to the following boundary conditions

\[
\text{Boundary conditions} \begin{cases} u(0, t) & \geq A[1 - \cos(2\pi ft)] \\ u(z, 0) & = 0, \quad \text{for } z > 0 \\ \partial_z u(z, 0) & = 0, \quad \text{for } z > 0 \\ \partial_z u(z, t) & \bigg|_{z=L} = 0 \end{cases} \tag{2}
\]

In our simulations, the projectile is released from the throwing engine as soon as the strain crosses from positive to negative at \( z = 0 \). Note that it is possible for a projectile to reconnect to the engine after takeoff, that is, the engine can catch up with trailing edge and thereby result in successive stages of ejection and reconnection.

**C. Spring-mass model**

We derive here the results obtained with a simple spring-mass projectile - a point mass \( m \) on top of a massless spring of stiffness \( k \) and free length \( L \) - to give insights about the role of inertia and elasticity in throwing experiments. This system has an eigenfrequency \( f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \).

![Figure 2](image_url)

**FIG. 2.** Energy transfer factor \( \alpha \) for the spring-mass model. This function \( \alpha \) reaches 3.13 for \( f_0/f = 1.62 \).
We denote \( z_m \) as the position of the point mass:

\[
\ddot{z}_m = -(2\pi f_0)^2 (z_m - z_p - L),
\]

where \( z_p = A(1 - \cos(2\pi f t)) \) measures the position of the engine plate. The solution of this forced harmonic oscillator equation, with \( z_m(0) = L \), is written:

\[
z_m(t) = L + A + \frac{A}{f^2 - f_0^2} \left( f_0^2 \cos(2\pi f t) - f^2 \cos(2\pi f_0 t) \right)
\]

The projectile takes off when \( z_m - z_p = L \), at time \( t_e = \frac{1}{f + f_0} \). The velocity ratio at takeoff \( \frac{V_c}{V_{p*}} \) is predicted to be:

\[
\frac{V_c}{V_{p*}} = \frac{\dot{z}_m(t_e)}{2\pi f A} = \frac{f_0/f}{f_0/f - 1} \sin \left( \frac{2\pi}{f_0/f + 1} \right).
\]

In Fig. 2, we show that the energy transfer factor \( \alpha = \left( \frac{V_c}{V_{p*}} \right)^2 \) reaches the maximum 3.13 for \( f_0/f \simeq 1.62 \). We interpret that this peak efficiency results from a particular match between the acceleration time of the engine \((\sim 1/f)\) and the typical time of the projectile deformation \((\sim 1/f_0)\). Finally we point out that this projectile has a maximum energy transfer factor 3.13 even higher than the maximal value 2.5 obtained with an homogeneous linear projectile [28].

III. RESULTS

We studied configurations where the rigid layer was on top and where the soft one was on top, and we varied the ratio \( x = L_s/L \) between the length of the soft layer \( L_s \) and the projectile length, \( L \).

A. Rigid-on-top configuration

We first consider the rigid-on-top configuration. In Fig. 3A,B we present the height reached by various projectiles for \( c_s = 3.4 \text{ m.s}^{-1} \), for several \( x \) values and for two different engine frequencies, 34 Hz and 76 Hz, leading to ratios \( c_s/Lf \) equal to 6.5 and 2.9, respectively. In all cases, there is a very good agreement between experiments and simulations. For both frequencies, the expected rigid behavior, \( \alpha = 1 \), is recovered when \( x \) equals 0. For intermediate values of \( x \), we observe a peak in the energy transfer factor. It is particularly pronounced for \( c_s/Lf = 2.9 \). In this case, \( \alpha \) reaches values close to 3 for both the simulations and experiments.

The existence of optima in \( \alpha \) was examined systematically with simulations, by varying the values of \( c_s/Lf \) and \( x \) in Eq. (1), and we present the results of this analysis in Fig. 3C,D,E. We note the existence of a best ejection performance region with \( \alpha \sim 3.1 \), which follows the curve \( c_s/Lf \sim 10\sqrt{x} \), from \( x = 0 \) until \( x \sim 0.3 \).

This maximal value reminds \( \alpha \sim 3.13 \) obtained with the spring-mass model in Sec. IIIC and we propose insights about this peculiar behavior: if the rigid part is assumed infinitely rigid, the system can be described by a single spring-mass system of mass \( m \sim SL\rho \) (the total mass) and of spring stiffness \( k = SE_s/L_s = SE_s/xL \) (the value of the soft part only), with \( S \) the area of the projectile cross section. Following the definition of \( f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \), the highest efficiency found for \( f_0/f = 1.62 \), we deduce that the position of the “optimal crest” can be determined from \( c_s/Lf = 1.62 \pi \sqrt{x} / 10.2 \sqrt{x} \), which gives a perfect match with respect to the predictions of the continuous model, as presented in Fig. 3C,D. The quasistatic assumption that the elasticity of the soft part can be accounted by an ideal homogeneous spring requires that the typical time of the ejection \((\sim 1/f)\) is
significantly larger than the propagation time in the soft layer ($\propto xL/c_s$). In the vicinity of the optimal ejection, $f \sim f_0 \sim \frac{1}{\sqrt{\frac{x}{c_s}}}$ and the condition becomes $\sqrt{x} \ll 1$, or simply $x \ll 1$. This is consistent with the experiment E2 in Fig.1C: the soft small layer seems to be deformed homogeneously as attested by the uniform coloring.

In Supplemental Material [29], we demonstrate that we need to add the condition $E_s/E_r \ll x$ to account for the finite rigidity of the top part and distinguish between $x = 0$ and the limit $x \to 0$. Given the values of $E_s/E_r$ (typically $10^{-6}$ in our experiments), this moderately thin soft layer limit $E_s/E_r \ll x \ll 1$, hereafter referred as $x \to 0$, is singular and differs from the rigid case $x = 0$: this system gives a best ejection $\alpha = 3.13$ for $c_s/Lf = 10.2\sqrt{x}$, in very good agreement with the simulations. $x = 0$ implies $\alpha$ equals one, as expected, because the projectile can be considered as rigid. The limit $x \to 0$ is of particular interest, because we show that the addition of a thin soft layer can lead to very efficient ejection if the layer properties match the dynamics of the catapult. For larger values of $x$, the crest follows $c_s/Lf = 6.8$ and $\alpha$ tends to 2.5, as shown in Fig.3D,E; the limit $x = 1$ recovers the results obtained with a soft homogeneous material [28].

\section{Soft-on-top configuration}

The soft-on-top configuration results are shown in Fig.4. Clearly the ejection performances are lower than in the previous case: the heights reached by these projectiles are notably smaller than the rigid-on-top experiments, for the two frequencies of the ejection engine, as shown in Fig.4A,B. For $c_s/Lf = 6.5$ (see Fig.4A), there is a smooth transition in the transfer factor from a rigid object ($x = 0$) to a soft homogeneous projectile ($x = 1$), and the highest values in $\alpha$ are reached near $x = 1$, that is, for soft homogeneous projectiles. Fig.4B shows that, in the case $c_s/Lf = 2.9$, there is no clear advantage in using a bi-layered projectile over a homogeneous one. In order to search the parameter space for the best ejection, we show in Fig.4C,D,E the numerical predictions of the model, and it appears that the best factor $\alpha$ is reached for $x = 1$ (i.e., a soft homogeneous projectile). The soft-on-top configuration (Fig.4C,D) seems systematically less efficient than the rigid-on-top one (Fig.3C,D).

\section{An optimal dynamics}

To summarize, rigid projectiles designed with an additional soft layer at their trailing edge exhibits an “optimal crest” that gives an optimal value for $\alpha$, whatever the value of $x$, if $c_s/Lf$ is adjusted correctly; it lies in the range $\alpha = 3.1$ for $x \to 0$ to $\alpha = 2.5$ for $x = 1$.

In terms of the dynamics, optimal ejection is provided by a tuned delayed response of the soft layer. To illustrate this idea, we compare in Fig.5, the temporal evolution of the relevant quantities during the ejection of a rigid projectile (A) and an optimal bi-layered projectile (B) given by the spring-mass model with $f_0/f = 1.62$ (Sec.II C). During the takeoff, the duration of the contact of the projectile with the plate is greatly enhanced in (B) with respect to (A). In the rigid case, the contact force $F$ decreases from an initial maximum towards zero as the plate reaches its maximum velocity $V_p^\star$. In addition, the maximal force exerted onto the projectile is greater in (B) than in (A). Since the transfer of energy from the plate to the projectile is given by $\int_0^{\infty} F(t) \dot{z}_p(t)dt$, where $\dot{z}_p(t) = 2\pi Af\sin(2\pi ft)$ is the plate velocity and $F(t)\dot{z}_p(t)$ is the instantaneous mechanical power, we infer that the rigid case cannot be optimal since the contact force and the plate velocity are never maximal at

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Ejection of projectiles with a soft top layer. We plot $\alpha$ vs $x$ for the engine frequencies 34 Hz and 76 Hz in (A) and (B), respectively. Images of the maximal heights are displayed for each case. The experimental measurements are shown with symbols, while the numerical predictions are represented by dashed curves. (C) Color plot of $\alpha$ as a function of $c_s/Lf$ and $x$ (color scale displayed on top), deduced from the numerical model. The particular values of $c_s/Lf$ 2.9 from (A) and 6.5 from (B), are represented by horizontal lines (using the color scheme of the two cases). (D) $c_s/Lf$ as a function of $x$ for the maximal $\alpha$-value found along a vertical line in the $\alpha$-diagram (C). (E) Maximal $\alpha$-value as a function of $x$.}
\end{figure}
the same time. In contrast, the bi-layered projectile delays the release of elastic energy and has a contact force in phase with the plate velocity, which ensures an optimal transfer. The nature of the contact force is the key element to discriminate projectiles. If we imagine an ideal engine that could prescribe the same time dependent force instead of a prescribed engine plate displacement, the motion would be directly dictated by Newton’s second law and the ejection velocity would be identical whatever the projectile.

**FIG. 5.** Temporal evolution of the plate (thin black curves) and projectile velocities (thin colored curves), the contact force (thick curves) and the power (dashed curves) for a rigid projectile (A) and an optimal bi-layered projectile (B). The energy transferred to the projectiles is represented by the area underneath the power curve. Dimensionless quantities are represented and velocity, force and time are divided by $2\pi Af$, $mA(2\pi f)^2$ and $1/f$, respectively.

### IV. DISCUSSION

In the light of the results, we can discuss the relevance of such projectiles for human throwing. Following Cross [30], the total kinetic energy provided by our body for an overarm throw is only partially transferred to the (rigid) projectile. In fact, segments of our arm, which can be equated with the throwing engine in our study, are still in motion once the projectile is ejected. Assuming that these segments move with the same velocity as that of the projectile, the throw efficiency (defined here as the ratio of the projectile kinetic energy to the kinetic energy provided by our body) is a function of the mass ratio $m/M$, where $m$ is the mass of the projectile and $M$ the equivalent mass of the throwing engine, which corresponds roughly to the mass of the forearm and hand, typically $M = 1 \sim 2$ kg [30]. In the case of heavy projectiles, such as shot put, when $m/M \gg 1$, the thrower can mobilize most of the kinetic energy for the projectile and the efficiency is close to one. Conversely for $m/M \ll 1$ this mode of propulsion becomes ineffective and most of the kinetic energy remains inside our body, leading to potential injuries. This could explain the difference between baseball and (American) football overarm throws. Both have very similar throwing sequences, but the difference in the mass of the balls, 140-150 g and 400-430 g respectively, is significant and makes the baseball pitcher more susceptible to injury than his football counterpart [31]. In the same vein, Cohen and Clanet [32] have shown that the mode of propulsion for ball sports (throwing vs hitting) correlates strongly with the mass ratio between the ball and the throwing instrument, which in the light of our results can be explained by a pursuit of the maximum energy transfer rate.

Actually we can experience the effect ourselves with two more extreme throwing examples: try to perform an overarm throw with a golf ball and a handball, 45 and 450 g respectively, as fast as you can. The throw with the heaviest ball feels smooth, while in the other case you have a frustrated impression that you cannot mobilize all your strength and that your arm gets overstretched.

Our study is particularly relevant for the inefficient limit $m/M \ll 1$, since the motion of the throwing engine is not influenced by the mass of the projectile, and we see a huge potential in designing projectiles to delay the release of elastic energy and increase the energy transfer by 300%. As a real-life example, we could imagine adding a homogeneous spring, with spring constant $k$, at the trailing edge of the (rigid) golf ball to increase its throw efficiency. Our study predicts that the peak efficiency is reached when $\sqrt{k/m} = 10.2f$ (Sec.II C). With a typical throw time $1/f \sim 1$ s, this leads to $k \sim 5$ N.m$^{-1}$. For heavy objects we would not expect a large difference in the efficiency of rigid and time-delayed projectiles, although a slight optimization of a few percent may still be relevant in the context of competition.

Finally, it is currently being debated how to regulate footwear in road running [33]. For example, footwear can be optimized to not only conserve energy but also to time the energy release from the sole according to the characteristics of the individual runner. To ensure fairness, it has been proposed to regulate the thickness of the sole [33], and thereby reduce the number of parameters one can use for optimization during the technological development. However, our experiments show (see for example Fig.3C) that a limitation on the thickness of the soft layer can potentially be compensated for by a change in the elastic properties.
V. CONCLUSION

By modifying the mechanical properties of a projectile, the throw efficiency can be significantly changed. For optimal throws, the key point is to delay the reaction force experienced at the thrower-projectile contact such that the contact force and velocity are in phase during the propulsive phase, thereby maximizing the mechanical power. For a homogeneous projectile or a spring-mass system, this is obtained by matching the throwing time with the eigenperiod of the projectile oscillations. For a bi-layered projectile, there is an optimum if the soft part is placed at the trailing edge. In this case, the gain in translation energy is a factor of 3 larger than for a rigid projectile. From an engineering point of view, massive propulsion engines are not the best choice to propel light projectiles. Actually, the lower the mass of the projectile, the lower the mass of the throwing instrument should be, in order to achieve the best efficiency. If the mass cannot be adjusted, projectiles designed for elastic-assisted throwing seem to be a very good option.

ACKNOWLEDGMENTS

We thank Fabrice Mortessagne for his support of the project, and acknowledge the financial support of CNRS. F.C. and C.R. thank Lorenzo Betti and Nicolas David for their input at the beginning of the project.